Asymmetry in Procurement Auctions:
Evidence from Snow Removal Contracts*

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Abstract

Differences in cost efficiency and productivity across firms may introduce asymmetries in procurement auctions. Relying on a structural approach, this paper investigates potential asymmetry among firms bidding for snow removal contracts in Montréal (Canada). The empirical results show that firms located in close proximity have a cost advantage relative to other firms in the most urbanized part of Montréal because of prohibitive equipment storage costs. The extent of inefficiency due to asymmetry is empirically assessed. Various policy experiments are performed. A bidding preference policy shows that the city could expect to reduce its costs for allocating snow removal contracts.

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1 Introduction

When bidders' willingness to pay depends in part on observable variables, the principal's interest is not best served by creating a level auction playing field. This paper studies procurement auctions for snow removal contracts in Montréal. It investigates the degree to which companies in disadvantageous locations should win even if they do not submit the lowest bid. From the submitted bids, a novel empirical methodology estimates the extent of cost variation across firms along with its variation within location. Although the dispersion is substantial, the gains from treating bidders unequally is limited.

When observed individual characteristics affect the firms' costs and their bidding strategies, all bidders cannot be treated alike. These differences in characteristics or asymmetries among bidders may arise from their size as noted by Laffont et al. (1995), their capacity constraints as in Jofre-Bonet and Pesendorfer (2003) or the possession of better information as in Hendricks and Porter (1988) and Hendricks et al. (1994). Other examples include collusion among a group of bidders as in Porter and Zona (1993), Baldwin et al. (1997), Pesendorfer (2000) and Bajari and Ye (2003), or joint bidding as in Hendricks and Porter (1992) and Campo et al. (2003). Asymmetry among bidders has also been acknowledged by economists in procurement auctions, when transportation of heavy equipment increases firms' costs as in Bajari (1997), domestic firms compete with foreign firms, etc. Within the private value paradigm, the theoretical auction literature has shown that the revenue equivalence theorem no longer holds with asymmetric bidders and that it might induce some inefficiency. See Klemperer (1999), Maskin and Riley (2000a) and Krishna (2002). It is generally beneficial for the auctioneer to exploit this

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asymmetry by giving a price preference to bidders who are in a weak position to win the auction. Some public institutions apply a price preference as high as 50% such as in defense contracts. See McAfee and McMillan (1989) and Krishna (2002) for the optimal mechanism design and Corns and Schotter (1999) for experimental evidence.

The City of Montréal in Canada organizes auctions to allocate snow removal contracts. Because of the equipment size and weather conditions, firms located relatively far from the snow removal location will have to rent a storage space for their equipment and trucks. This substantial additional cost may introduce some asymmetry among firms as it increases their costs. In particular, the distance between the firm and the contract location has been found to be a source of asymmetry among firms competing for asphalt coverage contracts by Bajari (1997). The firm facing this extra cost will have to bid more aggressively to compensate for its cost disadvantage relative to other firms that do not. Thus bidding strategies will be different across firms. It may also prevent the least efficient firms from participating to the auction as their bids will not be competitive. In this respect, it may act as a barrier to entry and potentially reduce the level of competition in the long run as noted by McAfee and McMillan (1989) and Harstad et al. (2003).

Auction theory provides a solid background to analyze such empirical phenomena. A major difficulty with asymmetric auction models is that no closed form for the equilibrium strategies can be obtained. This represents a major drawback for developing an econometric model since numerical methods are required. See Marshall et al. (1994) and Bajari (1997, 2001). We model the asymmetry among firms according to their location with respect to the location of the contract. Extending Guerre et al. (2000), we propose a convenient method to estimate the bidders' cost distributions. This method does not require solving for the bidding strategies and is computationally straightforward. The comparison of estimated equilibrium strategies allows us to assess the degree of asymmetry, while bidders' estimated costs allow us to assess the extent of inefficiency. Moreover, various policy experiments can be conducted such as the optimal mechanism design following Myerson (1981), an ascending auction and an auction with a price-preference policy under various alternatives. See McAfee and McMillan (1989), Branco (1994) and Kim (1994) for price-preference auctions, Harstad et al. (2003) for simulating a price-preference policy within a common value framework, Corns and Schotter (1999) for an experimental study of bidders' preference treatment and Ayres and Cramton (1996) for an empirical study of

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preference policies towards firms owned by minorities using the spectrum auction data.

The paper is organized as follows. Section 2 presents the snow clearance auction data and discusses some findings suggesting bidders' asymmetry. After a brief presentation of the asymmetric auction model, Section 3 focuses on the econometric model closely derived from the game theoretic auction model. The model identification is established and a two-step nonparametric procedure is derived to estimate bidders' latent cost densities. Section 4 presents the empirical results while a fifth section concludes.

2 The Snow Clearance Auction Data

2.1 The Contract and the Auction

The City of Montréal conducts first-price auctions to allocate five-year snow clearance contracts. The City provides a booklet to all interested firms with detailed information including a map of the area, the total length of the streets and the distance to the closest snow dump. It also lists requirements about the equipment to be used as well as other various obligations. Among the various factors that firms take into account, the length of the tract and the distance to the dump are two important characteristics. Longer tracts may induce smaller costs per meter as some scale economies are expected, while the distance to the dump may increase transportation costs. See Flambard et al. (2004). The contracts run from November 15th to March 15th. Firms submit a price per meter. The City of Montréal announces a reserve price above which it rejects any offer. The reserve price is computed by the office of public works, which estimates the cost for a firm with average productivity and cost. The firm submitting the lowest bid wins the auction and is paid a total of its bid times the length of the tract corrected for snow precipitations. Because of the precipitation variability, the City increases (decreases) the price by 0.4% for each centimeter of snow above (below) two hundred centimeters. If the total precipitation for the season is below one hundred centimeters, a minimum price is guaranteed and the price cannot fall below 60% of the bid value. Snow removal is often a secondary activity while firms' main activity is in construction, paving, excavating, landscaping or lawnmowing services.

2.2 The Bidding Data

A total of 61 auctions are held from 1990 to 1998 (no auction is held in 1994). Sixty different firms participate in these auctions. For each auction, the data provide the tract

length in meters, the distance to the snow dump in kilometers, the exact location, the reserve price per meter, the submitted bids per meter, both expressed in constant 1986 Canadian dollars, the number of bidders and their identity. A total of 457 bids are collected. Summary statistics are given in Table 1. The auctions are quite competitive with an average number of active participants varying from 5 to 11. We observe a downward trend in bids, winning bids and reserve prices over time. Bids and reserve prices have decreased on average by 26% and by 31%, respectively. A discussion with the professional association of snow removers reveals that the equipment has improved and can be used for several purposes. Firms have enjoyed scope economies due to their multiple activities and productivity gains caused by more efficient equipment. The tract length has also increased allowing firms to benefit from scale economies given their important fixed costs. An increased distance to the dump may have, however, counterbalanced these cost reductions. We find a negative correlation between bids and length (-0.49) and a positive correlation between bids and distance (0.40). The decline in bids suggests that both technological progress and economies of scale outweigh the additional costs due to an increased distance to the dump. Decreasing bids could also be explained by a more intense competition. We do not observe, however, any particular trend in competition.

Reserve prices and bids are highly correlated (0.83). Except for 1990 and 1993, the winning bid is on average 11.2% to 17.4% less than the reserve price suggesting a binding reserve price. When estimating nonparametrically the probability that the bids are in the neighbourhood of the reserve price, i.e. $\Pr((1 - \delta)p_0 \leq b \leq p_0)$ for an average tract, we find this probability equal to 0.19 for $\delta = 0.05$, 0.33 for $\delta = 0.10$ and 0.51 for $\delta = 0.2$. This clearly shows that the reserve price truncates the bid distribution and thus acts as an effective screening device for participating to the auction. As a result, the number of potential bidders may be larger than the observed number of participants.

2.3 Some Evidence of Asymmetry Among Bidders

Snow removal involves transportation of heavy equipment such as trucks, snowplowers and sweepers leading the firms to rent a storage space. This may represent an additional cost in the urbanized areas of the city. The City of Montréal is located on an island between two rivers. The west half of Montréal is highly urbanized while the east half is more industrialized. Tract characteristics may differ according to their west/east location.

Firms are mainly located in east Montréal or in the suburbs off of the island. The firms in east (west) Montréal may have a relative cost advantage on eastern (western) tracts because of their proximity. Renting a storage space will not be absolutely necessary for these firms. This would suggest a possible asymmetry among firms due to their location with respect to the location of the tract. The purpose of this section is to empirically investigate firms' potential asymmetry in terms of their proximity to the tract. The exact distance of the firm to the tract is not available. Above a given distance, firms have to rent a space whatever their distance to the tract is. Thus, it seems more appropriate to study asymmetry in terms of location than in terms of distance. See Bajari (1997) for a study of asymmetry in terms of firms' distance for asphalt coverage contracts. Likewise, some firms may handle several tracts giving rise to asymmetric capacity constraints as in Jofre-Bonet and Pesendorfer (2003). Because snow removal is a secondary activity to supplement income, capacity constraints do not seem, however, to be a major issue in these auctions.

Tables 2 and 3 provide summary statistics for the 32 eastern tracts and the 29 western tracts. Bids, winning bids and reserve prices are on average slightly larger for eastern tracts than for western tracts. The number of active participants is, however, quite different. The competition for western tracts is about 17% less on average than for eastern tracts. This could be explained by the large number of firms in east Montréal. We consider three types of participating firms according to their location, namely east, west and off of the island. Almost five east firms participate on average to east tract auctions while only two participate to west tract auctions. Similarly, almost three west firms submit bids on western tracts while only one submits a bid on eastern tracts on average. The participation of firms located off of the island is quite similar across tracts. This suggests that firms are more likely to participate to auctions of tracts located in their area. These differences in participation are evidence of asymmetry among firms.

To further assess the presence of asymmetry, we propose to check whether some bidders are more likely to win the auctions than others. We compare the firms' participation rate with their probabilities of winning the auction. Out of the 261 bids submitted on eastern tracts, 57% are submitted by east firms, 18% by west firms and 25% by firms located off of the island, while 72% of eastern tracts are won by east firms, 12% by west firms and 16% by outside firms. The difference is significant for east firms. For outside firms it is barely

significant, while it is not for west firms. Out of the 196 bids submitted on western tracts, 26% are submitted by east firms, 41% by west firms and 33% by firms located off of the island, while 14% of western tracts are won by east firms, 62% by west firms, and 24% by outside firms. The difference is significant for east and west firms, while it is barely significant for outside firms. Asymmetry seems to prevail on western tracts, while it is negligible on eastern tracts. West firms have a relative cost advantage on western tracts and are more likely to win these tracts than other firms. The low participation of east firms on western tracts and of west firms on eastern tracts suggests that their possibly larger costs prevent them from participating to the auctions.

Another empirical assessment of asymmetry can be conducted through a regression of the logarithm of the bids on the number of bidders, the length of the tract, the distance to the dump and a time trend. The idea is to test whether this causal relationship differs with the firm's type. A similar method has been used in Pesendorfer (2000) to detect collusion in school milk auctions. Considering the 262 bids in east tract auctions, we run three separate regressions for the 150, 46 and 65 bids submitted by east, west and outside firms, respectively. A Chow test shows that coefficients for west and outside firms are not significantly different. We then assume that west and outside firms can be treated as alike on eastern tracts. We consider two separate regressions for the bids submitted by east firms and other firms. A Chow test shows equal coefficients between the two groups. We apply a similar method for bids submitted in west tract auctions. East and outside firms can be pooled in a single group. By considering the two groups on western tracts, a Chow test shows different coefficients. These results suggest that asymmetry is nonexistent on eastern tracts, while asymmetry is potentially present on western tracts.

Finally, we estimate the bid cumulative density functions. If there is asymmetry, the bid distribution of strong bidders should stochastically dominate the one of weak bidders. Hereafter, strong bidders are firms with a relative cost advantage (lower cost) and are more likely to win the auctions. Similarly, weak bidders are firms with a relative cost disadvantage (larger cost) and are less likely to win the auctions. For a median tract, Figures 1 and 2 display the empirical bid distributions for weak and strong bidders in western and eastern tracts, respectively. On western tracts, the bid distribution of west (strong) firms is above the bid distribution of other (weak) firms as shown by Figure 1. We do not observe such dominance on eastern tracts in Figure 2. A Smirnov-Kolmogorov

test for stochastic dominance has been performed. There is strong rejection on eastern tracts while there is weak rejection on western tracts.

Comparisons of firms' participation, firms' probabilities of winning, firms' bidding behavior as well as bid distributions suggest that there is some asymmetry on western tracts while this asymmetry is nonexistent on eastern tracts.

3 The Model, its Identification and Estimation

This section briefly presents an asymmetric auction model with independent costs and a binding reserve price. Conditional cost independence is a reasonable approximation because firms have different opportunity costs resulting from their different main activities. In procurement auctions, Bajari (1997), Jofre-Bonet and Pesendorfer (2003) and Krasnokutskaya (2002) have also considered independence. Though a common component may exist, we consider it as negligible in this activity. Subcontracting is allowed for snow transportation but not for snow clearance. See Li et al. (2000, 2002) and Campo et al. (2003) for the estimation of the affiliated private value model for symmetric and asymmetric bidders, respectively. The identification of the model and a nonparametric estimation procedure are discussed.

3.1 An Asymmetric Auction Model

A single and indivisible project is auctioned to n risk neutral bidders. Though our results can be generalized, we consider two types of bidders as in Maskin and Riley (2000a). Group 1 (0) refers to strong (weak) bidders who are (not) located in proximity to the tract. Types 1 and 0 consist of n_1 and n_0 bidders, respectively, with $n_1 + n_0 = n$ and $n \geq 2$. Let $c_{1i}, i = 1, \ldots, n_1$ and $c_{0i}, i = 1, \ldots, n_0$ denote the bidder's cost in group 1 and 0. The costs c_{1i} and c_{0i} are drawn independently from the distributions $F_1(\cdot)$ and $F_0(\cdot)$, respectively, defined on $[\underline{c}, \overline{c}]$. The costs are all independent. Both distributions are continuous with densities $f_0(\cdot)$ and $f_1(\cdot)$. We consider a first-price sealed-bid auction in which the buyer announces a reserve price $p_0 \leq \overline{c}$ and rejects any bid above this value. The number of actual participants may differ from the number of potential bidders n as some bidders may have a cost larger than p_0. Bidders know their own costs as well as $[F_0(\cdot), F_1(\cdot), n_0, n_1]$.

At the Bayesian Nash equilibrium, each bidder i of type 1 chooses his bid b_{1i} to maximize his expected profit $E[(b_{1i} - c_{1i})\mathbb{1}(b_{1i} \leq B_{1i})|c_{1i}, c_{1i} \leq p_0]$, where $B_{1i} = \min(s_0(C_0), s_1$

(C_{1i}^*), with $C_0 = \min_i c_{0i}$, $C_{1i}^* = \min_{j \neq i} c_{1j}$, $s_0(\cdot)$ and $s_1(\cdot)$ are the strictly increasing equilibrium strategies adopted by bidders of type 0 and 1, respectively. This gives

$$\max_{b_{1i}}(b_{1i} - c_{1i})\Pr[s_1^{-1}(b_{1i}) \leq C_{1i}^* \text{ and } s_0^{-1}(b_{1i}) \leq C_0], \qquad (1)$$

if $c_{1i} \leq p_0$, where $s_j^{-1}(\cdot)$ denotes the inverse of the equilibrium strategy. The probability is $[1 - F_1(s_1^{-1}(b_{1i}))]^{n_1-1}[1 - F_0(s_0^{-1}(b_{1i}))]^{n_0}$. Each bidder i of type 0 chooses his bid b_{0i} to maximize his expected profit $\mathrm{E}[(c_{0i} - b_{0i})\,\mathbb{1}(b_{0i} \leq B_{0i})|c_{0i}, c_{0i} \leq p_0]$, where $B_{0i} = \min(s_0(C_{0i}^*), s_1(C_1))$, with $C_{0i}^* = \min_{j \neq i} c_{0j}$, $C_1 = \min_i c_{1i}$. This gives

$$\max_{b_{0i}}(b_{0i} - c_{0i})\Pr[s_0^{-1}(b_{0i}) \leq C_1 \text{ and } s_0^{-1}(b_{0i}) \leq C_{0i}^*], \qquad (2)$$

if $c_{0i} \leq p_0$, where the probability is $[1 - F_1(s_1^{-1}(b_{0i}))]^{n_1}[1 - F_0(s_0^{-1}(b_{0i}))]^{n_0-1}$.

Differentiating (1) and (2) with respect to b_{1i} and b_{i2} gives the following system of first-order differential equations defining the equilibrium strategies $s_1(\cdot)$ and $s_0(\cdot)$

$$(b_{1i}-c_{1i})\left[(n_1-1)\frac{f_1(s_1^{-1}(b_{1i}))}{1-F_1(s_1^{-1}(b_{1i}))}\frac{1}{s_1'(s_1^{-1}(b_{1i}))}+n_0\frac{f_0(s_0^{-1}(b_{1i}))}{1-F_0(s_0^{-1}(b_{1i}))}\frac{1}{s_0'(s_0^{-1}(b_{1i}))}\right]=1, \quad (3)$$

$$(b_{0i}-c_{0i})\left[n_1\frac{f_1(s_1^{-1}(b_{0i}))}{1-F_1(s_1^{-1}(b_{0i}))}\frac{1}{s_1'(s_1^{-1}(b_{0i}))}+(n_0-1)\frac{f_0(s_0^{-1}(b_{0i}))}{1-F_0(s_0^{-1}(b_{0i}))}\frac{1}{s_0'(s_0^{-1}(b_{0i}))}\right]=1, \quad (4)$$

for $c_{1i} \leq p_0$, $c_{0i} \leq p_0$, where $b_{1i} = s_1(c_{1i})$ and $b_{0i} = s_0(c_{0i})$, subject to the boundary conditions $s_1(p_0) = s_0(p_0) = p_0$ and $s_1(\underline{c}) = s_0(\underline{c})$ with $p_0 \leq \overline{c}$. For the existence and uniqueness of the equilibrium, see Maskin and Riley (1996, 2000b) and Lebrun (1996, 1999). This system is quite complex and intractable. Marshall et al. (1994) propose some numerical algorithms for solving this system, while Bajari (1997, 2001) suggests some numerical procedures within a Bayesian estimation context. The necessity to use complex algorithms has been a major drawback for analyzing asymmetric auction data. In particular, because this system does not have an explicit solution, simulation-based methods proposed by Laffont et al. (1995) are impossible to implement. In the following subsections, we propose a simple and convenient method to circumvent this difficulty extending Guerre et al. (2000) results.

The estimation results will allow us to conduct some policy experiments. For this purpose, we review some theoretical results on asymmetric auctions. First, the revenue

equivalence theorem no longer holds with asymmetric bidders. Asymmetries among bidders do not affect bidding behavior in ascending or second-price auctions, i.e. it is still a weakly dominant strategy for each bidder to bid his own cost. Because a closed form for the bidding strategies is not available in first-price auctions, comparisons are rather difficult to make and no general ranking can be obtained. See Krishna (2002). Cantillon (2004) also finds that asymmetric auctions tend to lower the expected revenue formalizing the idea that competition is reduced by bidders' asymmetries.

Second, first-price auctions may lead to inefficient allocations, while ascending auctions always provide efficient allocations. The intuition is quite simple. In a first-price auction, at a given cost, the weak bidder will bid more aggressively than the strong bidder to overcome its relative disadvantage. As a result, the contract may not be allocated to the bidder with the lowest cost. Third, the buyer may gain in exploiting the bidders' asymmetries while implementing discriminatory auctions. Discriminatory auctions under the form of a price-preference policy are frequently observed. Recent examples include the spectrum auctions held by the Federal Communications Commission that made use of various discriminatory policies in the spirit of affirmative action. See Ayres and Cramton (1996).

Following Myerson (1981), the optimal mechanism design with asymmetric bidders is as follows. See also Krishna (2002). It calls for the buyer to set discriminatory reserve prices. The reserve price for strong bidders is the solution of $p_1^* = c_0 - F_1(p_1^*)/f_1(p_1^*)$, while c_0 denotes the buyer's value to perform the contract. Similarly, the reserve price for weak bidders solves $p_0^* = c_0 - F_0(p_0^*)/f_0(p_0^*)$. Let us consider the virtual costs $J_1(c) = c + F_1(c)/f_1(c)$ and $J_0(c) = c + F_0(c)/f_0(c)$ for any cost c. The optimal mechanism in terms of expected cost for the buyer is to allocate the contract to the bidder with the lowest value $J_i(c_{ij})$, $i = 0, 1$, $j = 1, \ldots, n_i$ and the winner pays $b_w = \sup\{c_{ij} : J_i(c_{ij}) \geq 0 \text{ and } \forall k \neq j, \forall k' = 0, 1, J_i(c_{ij}) \leq J_{k'}(c_{k'k})\}$. With such allocation and payment rules, it is in the best interest of the bidders to reveal their costs. This optimal mechanism discriminates in favor of weak bidders since the contract is awarded on the basis of virtual costs. The weak bidders will win more often than in the standard first-price auctions.

McAfee and McMillan (1989) consider a discounting function $z(\cdot)$ such that a weak bidder with cost c_0 will win against a strong bidder with cost c_1 if $z(c_0) < c_1$. They show that if both distributions are related by a spread-preserving change in mean, i.e.

$F_0(c + a) = F_1(c)$ for some $a > 0$, then the buyer should always discriminate in favor of the weak bidders. Their simulation study shows that the optimal preferential treatment should involve about one third of the cost difference between weak and strong firms.

In view of these results, we empirically assess the inefficiency and conduct policy experiments of the ascending auction, the optimal mechanism design with discriminatory reserve prices as well as a subsidy policy in favor of the weak bidders with various scenarios for the cost of public funds. As an illustration, the appendix contains a simulation study of such a subsidy policy.

3.2 Nonparametric Identification

Following the structural approach, observed bids are the equilibrium bids. The econometric model is closely derived from the model defined by the structure $[F_1(\cdot), F_0(\cdot)]$ and the system of differential equations (3)-(4). The structural econometric model is given by $b_{1i} = s_1(c_{1i}, F_1, F_0, n_1, n_0, p_0)$, $i = 1, \ldots, n_1$ and $b_{0i} = s_0(c_{0i}, F_1, F_0, n_1, n_0, p_0)$, $i = 1, \ldots, n_0$. In a first-price sealed-bid auction with a reserve price, the bids and the number of actual bidders are typically observed while the bidders' costs, their distributions and the numbers of potential bidders are not. Because costs are random, bids are naturally random. Let $G_1(\cdot)$ and $G_0(\cdot)$ be the corresponding bid distributions. With a binding reserve price, we need to consider the truncated distributions $G_1^*(\cdot| \cdot \leq p_0)$ and $G_0^*(\cdot| \cdot \leq p_0)$. Note that $G_1^*(b_{1i}) = F_1[s_1^{-1}(b_{1i})]/F_1(p_0)$ and $G_0^*(b_{0i}) = F_0[s_0^{-1}(b_{0i})]/F_0(p_0)$ with densities $g_1^*(b_{1i}) = f_1[s_1^{-1}(b_{1i})]/[F_1(p_0)s_1'(s_1^{-1}(b_{1i}))]$ and $g_0^*(b_{0i}) = f_0[s_0^{-1}(b_{0i})]/[F_0(p_0)s_0'(s_0^{-1}(b_{0i}))]$. The system of differential equations (3)-(4) can be rewritten as

$$c_{1i} = b_{1i} - \frac{1}{(n_1 - 1)\frac{g_1^*(b_{1i})F_1(p_0)}{1-G_1^*(b_{1i})F_1(p_0)} + n_0\frac{g_0^*(b_{1i})F_0(p_0)}{1-G_0^*(b_{1i})F_0(p_0)}} \equiv \xi_1(b_{1i}, G_0^*, G_1^*, n, F_0(p_0), F_1(p_0)), \quad (5)$$

$$c_{0j} = b_{0j} - \frac{1}{n_1\frac{g_1^*(b_{0j})F_1(p_0)}{1-G_1^*(b_{0j})F_1(p_0)} + (n_0 - 1)\frac{g_0^*(b_{0j})F_0(p_0)}{1-G_0^*(b_{0j})F_0(p_0)}} \equiv \xi_0(b_{0j}, G_0^*, G_1^*, n, F_0(p_0), F_1(p_0)), \quad (6)$$

for $i = 1, \ldots, n_1^*$ and $j = 1, \ldots, n_0^*$. Note that the system (5)-(6) can also be derived from (1)-(2) with probabilities expressed in terms of bid distributions.

The problem of identification reduces to whether $F_1(\cdot)$, $F_0(\cdot)$, n_1 and n_0 are identified from the observed bids $(b_{11}, \ldots, b_{1n_1^*}, b_{01}, \ldots, b_{0n_0^*})$ and the number of actual bidders n_1^* and n_0^*. The binding reserve price complicates the identification problem since (5) and (6) depend on n_1, n_0, $F_1(p_0)$ and $F_0(p_0)$, which are unknown. Our result complements

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Proposition 7 of Laffont and Vuong (1996), which states that the asymmetric IPV model with no reserve price is identified. Another interesting question is whether the model imposes any restriction on observables that can be used to test the validity of the model. Proposition 1 gives such restrictions on the bid distributions, the actual numbers of bidders and the inverse equilibrium strategies to be rationalized by an asymmetric model. The proof of Proposition 1 is given in the appendix.

Proposition 1: *Let $G_1^*(\cdot)$ and $G_0^*(\cdot)$ on $[\underline{c}, p_0]$, and $\pi_1(\cdot)$ and $\pi_0(\cdot)$ be discrete distributions. There exist $F_1(\cdot)$, $F_0(\cdot)$, n_1 and n_0, such that (i) $G_1^*(\cdot)$ and $G_0^*(\cdot)$ are the truncated bid distributions in a first-price sealed-bid auction with $p_0 \in (\underline{c}, \overline{c})$, and (ii) $\pi_1(\cdot)$ and $\pi_0(\cdot)$ are the distributions of n_1^* and n_0^*, respectively, if and only if the following conditions hold*

C1: $\pi_1(\cdot)$ and $\pi_0(\cdot)$ are Binomial with parameters $(n_1, F_1(p_0))$ and $(n_0, F_0(p_0))$, respectively, where $0 < F_1(p_0) < 1$ and $0 < F_0(p_0) < 1$,

C2: $b_{1i}, i = 1, \ldots, n_1^*$ and $b_{0i}, i = 1, \ldots, n_0^*$ are i.i.d. as $G_1^*(\cdot)$ and $G_0^*(\cdot)$ conditionally upon n_1^* and n_0^*, respectively, and $\lim_{b \uparrow p_0} g_1^*(b) = \lim_{b \uparrow p_0} g_0^*(b) = +\infty$,

C3: $\xi_1(b_{1i}, G_0^*, G_1^*, n, F_0(p_0), F_1(p_0))$ and $\xi_0(b_{0i}, G_0^*, G_1^*, n, F_0(p_0), F_1(p_0))$ defined in (5)-(6) are strictly increasing on $[\underline{c}, p_0]$ and their inverses are differentiable on $[\underline{c}, p_0]$.

If conditions C1–C3 hold, n_1, n_0, $F_0(p_0)$ and $F_1(p_0)$ are unique while $F_1(\cdot)$ and $F_0(\cdot)$ are uniquely defined on $[\underline{c}, p_0]$ as $F_1(\cdot) = F_1(p_0)G_1^*(\xi_1^{-1}(\cdot, G_1^*, G_0^*, n, F_1(p_0), F_0(p_0)))$ and $F_0(\cdot) = F_0(p_0)G_0^*(\xi_0^{-1}(\cdot, G_1^*, G_0^*, n, F_1(p_0), F_0(p_0)))$. Moreover, $\xi_1(\cdot, G_0^*, G_1^*, n, F_0(p_0), F_1(p_0))$ and $\xi_0(\cdot, G_0^*, G_1^*, n, F_0(p_0), F_1(p_0))$ are the quasi inverses of $s_1(\cdot, F_0, F_1, p_0, n)$ and $s_0(\cdot, F_0, F_1, p_0, n)$.

Proposition 1 extends Theorem 4 of Guerre <u>et al.</u> (2000) to asymmetric bidders. First, it shows that the model imposes some restrictions on the observables. Observed bids have to be mutually independent and the functions $\xi_1(\cdot)$ and $\xi_0(\cdot)$ have to be strictly increasing. If the latter are found to be increasing, then bidders have adopted the Bayesian Nash equilibrium strategies as defined by the model. Second, it shows that the numbers of potential bidders are identified from the numbers of actual bidders, while the latent cost distributions $F_1(\cdot)$ and $F_0(\cdot)$ are both nonparametrically identified on $[\underline{c}, p_0]$.

Though the existence and uniqueness of the equilibrium has been shown, Proposition 1 provides an empirical response to the problems of existence and uniqueness of the equilibrium. The validation of the restrictions C1–C3 on observables shows that the equilibrium

of the corresponding auction game exists. Similarly, the unique mapping between the observables and the model primitives assesses the uniqueness of the equilibrium.

The knowledge of $G_1^*(\cdot)$, $G_0^*(\cdot)$, $g_1^*(\cdot)$, $g_0^*(\cdot)$, n_0, n_1, $F_1(p_0)$ and $F_0(p_0)$ determines the costs c_1 and c_0 for any bid value. Our result avoids the use of complex algorithms to solve the system of differential equations (3)-(4) for determining the equilibrium strategies.

3.3 Estimation Method

The purpose of the structural approach is to estimate the underlying distributions $F_1(\cdot)$ and $F_0(\cdot)$. If one knows the functions and numbers involved in (5)-(6), one can compute the costs and estimate $f_1(\cdot)$ and $f_0(\cdot)$. It suggests a two-step estimation procedure. To take fully advantage of the nonparametric nature of our identification result, we prefer to use nonparametric estimators. We consider L auctions indexed by ℓ, $\ell = 1, \ldots, L$. Let z_ℓ be a vector of variables characterizing the auctioned object with $z_\ell \in I\!\!R^d$ and $n_{1\ell}$ and $n_{0\ell}$, $n_{1\ell} + n_{0\ell} = n_\ell$, be the numbers of potential bidders in each auction. We assume that all the information characterizing the auctioned objects is available to the analyst. For unobserved heterogeneity, see Krasnokutskaya (2002). We observe $\{b_{1p\ell}, p = 1, \ldots, n_{1\ell}^*, b_{0q\ell}, q = 1, \ldots, n_{0\ell}^*, p_{0\ell}, z_\ell, \ell = 1, \ldots, L\}$. We assume that $n_{1\ell}$ and $n_{0\ell}$ are constant across some known subsets of auctions. Moreover, $p_{0\ell}$ is an unknown deterministic function of z_ℓ and is bounded away from $\underline{c}(z_\ell)$ and $\overline{c}(z_\ell)$.

The model is conditional on the vector z_ℓ, the reserve price $p_{0\ell}$ and the number of bidders $n_{1\ell}$ and $n_{0\ell}$. Since $n_{1\ell}$ and $n_{0\ell}$ are constant across auctions and $p_{0\ell} = h(z_\ell)$, the model can be written conditionally upon z_ℓ only. The truncated bid distributions in the ℓ-th auction become the conditional distributions $G_1^*(\cdot|z_\ell)$ and $G_0^*(\cdot|z_\ell)$. Similarly, the cost distributions are the conditional distributions $F_1(\cdot|z_\ell)$ and $F_0(\cdot|z_\ell)$. As such, the cost distributions are varying across auctions through z_ℓ. For $p = 1, \ldots, n_{1\ell}^*$, $q = 1, \ldots, n_{0\ell}^*$ and $\ell = 1, \ldots, L$, (5) and (6) become

$$c_{1p\ell} = b_{1p\ell} - \frac{1}{(n_1 - 1)\frac{g_1^*(b_{1p\ell}|z_\ell)\phi_1(z_\ell)}{1-G_1^*(b_{1p\ell}|z_\ell)\phi_1(z_\ell)} + n_0\frac{g_0^*(b_{1p\ell}|z_\ell)\phi_0(z_\ell)}{1-G_0^*(b_{1p\ell}|z_\ell)\phi_0(z_\ell)}}, \tag{7}$$

$$c_{0q\ell} = b_{0q\ell} - \frac{1}{n_1\frac{g_1^*(b_{0q\ell}|z_\ell)\phi_1(z_\ell)}{1-G_1^*(b_{0q\ell}|z_\ell)\phi_1(z_\ell)} + (n_0 - 1)\frac{g_0^*(b_{0q\ell}|z_\ell)\phi_0(z_\ell)}{1-G_0^*(b_{0q\ell}|z_\ell)\phi_0(z_\ell)}}, \tag{8}$$

where $\phi_1(z_\ell) \equiv F_1(p_{0\ell}|z_\ell)$ and $\phi_0(z_\ell) \equiv F_0(p_{0\ell}|z_\ell)$.

Using observed bids, we can estimate nonparametrically $G_1^*(\cdot|\cdot)$, $G_0^*(\cdot|\cdot)$, $g_1^*(\cdot|\cdot)$ and

$g_0^*(\cdot|\cdot)$. Note that $G_1^*(\cdot|z_\ell) = G_1^*(\cdot, z_\ell)/f_z(z_\ell)$ and $G_0^*(\cdot|z_\ell) = G_0^*(\cdot, z_\ell)/f_z(z_\ell)$, where $f_z(\cdot)$ denotes the marginal density of z. Similarly, $g_1^*(\cdot|z_\ell) = g_1^*(\cdot, z_\ell)/f_z(z_\ell)$ and $g_0^*(\cdot|z_\ell) = g_0^*(\cdot, z_\ell)/f_z(z_\ell)$. We propose the following nonparametric estimators

$$\hat{G}_1^*(b, z) = \frac{1}{L h_G^d} \sum_{\ell=1}^{L} \frac{1}{n_{1\ell}^*} \sum_{p=1}^{n_{1\ell}^*} \mathbb{1}(b_{1p\ell} \leq b) K_G\left(\frac{z - z_\ell}{h_{1G}}\right),$$

$$\hat{G}_0^*(b, z) = \frac{1}{L h_G^d} \sum_{\ell=1}^{L} \frac{1}{n_{0\ell}^*} \sum_{q=1}^{n_{0\ell}^*} \mathbb{1}(b_{0q\ell} \leq b) K_G\left(\frac{z - z_\ell}{h_{0G}}\right),$$

$$\hat{g}_1^*(b, z) = \frac{1}{L h_g^{d+1}} \sum_{\ell=1}^{L} \frac{1}{n_{1\ell}^*} \sum_{p=1}^{n_{1\ell}^*} K_g\left(\frac{b - b_{1p\ell}}{h_{1g}}, \frac{z - z_\ell}{h_{1g}}\right),$$

$$\hat{g}_0^*(b, z) = \frac{1}{L h_g^{d+1}} \sum_{\ell=1}^{L} \frac{1}{n_{0\ell}^*} \sum_{q=1}^{n_{0\ell}^*} K_g\left(\frac{b - b_{0q\ell}}{h_{0g}}, \frac{z - z_\ell}{h_{0g}}\right),$$

$$\hat{f}_z(z) = \frac{1}{L h_z^d} \sum_{\ell=1}^{L} K_z\left(\frac{z - z_\ell}{h_z}\right),$$

for any value (b, z), where $\mathbb{1}(\cdot)$ is the indicator function, $K_G(\cdot)$, $K_g(\cdot)$ and $K_z(\cdot)$ are some kernels defined on compact supports, and h_{1G}, h_{0G}, h_{1g}, h_{0g} and h_z are some smoothing parameters called bandwidths. A discussion on the choice of kernels and bandwidths can be found in the appendix.

The pseudo costs can be estimated using (7) and (8) provided one can estimate n_1, n_0, $\phi_1(z_\ell)$ and $\phi_0(z_\ell)$. Natural estimators for n_1 and n_0 are $\hat{n}_1 = \max_\ell n_{1\ell}^*$ and $\hat{n}_0 = \max_\ell n_{0\ell}^*$. Note that this estimator can be generalized for n_1 and n_0 being constant on some subsets of auctions. Using $\mathrm{E}(n_{1\ell}^*|z_\ell) = n_1\phi_1(z_\ell)$ and $\mathrm{E}(n_{0\ell}^*|z_\ell) = n_0\phi_0(z_\ell)$, and solving for $\phi_1(z_\ell)$ and $\phi_0(z_\ell)$, we obtain for any value z

$$\hat{\phi}_1(z) = \frac{1}{\hat{n}_1 L h_z^d \hat{f}_z(z)} \sum_{\ell=1}^{L} n_{1\ell}^* K_z\left(\frac{z - z_\ell}{h_z}\right), \hat{\phi}_0(x) = \frac{1}{\hat{n}_0 L h_z^d \hat{f}_z(z)} \sum_{\ell=1}^{L} n_{0\ell}^* K_z\left(\frac{z - z_\ell}{h_z}\right) \quad (9)$$

using a kernel estimator for the nonparametric regression. Using (7) and (8), the pseudo costs can be estimated by replacing all the unknown functions and numbers by their estimated counterparts. This gives $\{\hat{c}_{1p\ell}, \hat{c}_{1q\ell}, p = 1, \ldots, n_{1\ell}^*, q = 1, \ldots, n_{0\ell}^*, \ell = 1, \ldots, L\}$. Because of well-known boundary effects in kernel estimation, costs may not be well estimated near the boundaries. This effect may be corrected by a trimming.

In the second step, this sample can be used to estimate the truncated densities $f_1^*(c|z)$

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and $f_0^*(c|z)$ by $\hat{f}_1^*(c|z) = \hat{f}_1^*(c,z)/\hat{f}_z(z)$ and $\hat{f}_0^*(c|z) = \hat{f}_0^*(c,z)/\hat{f}_z(z)$, where

$$\hat{f}_1^*(c,z) = \frac{1}{Lh_{1f}^{d+1}} \sum_{\ell=1}^{L} \frac{1}{n_{1\ell}^*} \sum_{p=1}^{n_{1\ell}^*} K_f \left(\frac{c - \hat{c}_{1p\ell}}{h_{1f}}, \frac{z - z_\ell}{h_{1f}} \right),$$

$$\hat{f}_0^*(c,z) = \frac{1}{Lh_{0f}^{d+1}} \sum_{\ell=1}^{L} \frac{1}{n_{0\ell}^*} \sum_{q=1}^{n_{0\ell}^*} K_f \left(\frac{c - \hat{c}_{0q\ell}}{h_{0f}}, \frac{z - z_\ell}{h_{0f}} \right),$$

for any value (c, z), where $K_f(\cdot)$ is a multivariate kernel, h_{1f} and h_{0f} are some bandwidths. Because the pseudo costs are always below the reserve prices, it introduces a truncation. Thus, the densities $f_1(c|z)$ and $f_0(c|z)$ can be estimated by

$$\hat{f}_1(c|z) \equiv \hat{\phi}_1(z)\hat{f}_1^*(c|z), \quad \hat{f}_0(c|z) \equiv \hat{\phi}_0(z)\hat{f}_0^*(c|z),$$

where $\hat{\phi}_1(z)$ and $\hat{\phi}_0(z)$ are estimated according to (9).

Using similar arguments and assumptions as in Guerre <u>et al.</u> (2000), our two-step estimators are uniformly consistent using appropriate bandwidths. The comparison of $\hat{\xi}_1(\cdot)$ and $\hat{\xi}_0(\cdot)$ as well as $\hat{f}_1(\cdot|\cdot)$ and $\hat{f}_0(\cdot|\cdot)$ will allow us to assess whether bidders are asymmetric. Because our estimation method is fully nonparametric, such information is revealed by the data and is robust to misspecification. Our method is straightforward and quick to implement. Since we recover bidders' costs, we can estimate the winner's rent by computing the differences between the winner's bid and cost. Similarly, the estimated densities can be used for policy purposes to simulate other auction mechanisms. Nonparametric estimators require, however, a substantial number of observations. Our data set provides a large number of bids, but a rather small number of auctions. The next section shows that we obtain good results given our sample sizes.

4 Asymmetry in Snow Clearance Auctions

4.1 Empirical Results

Considering the median value of the characteristic z, Figure 3 displays the estimated equilibrium strategies for western tracts, while Figure 4 displays their counterparts for eastern tracts. Note that $\hat{n}_{1W} = 6$, $\hat{n}_{0W} = 11$, $\hat{n}_{1E} = 9$ and $\hat{n}_{0E} = 6$. The variable z is constructed using an aggregation method from the distance to the snow dump, the length of the tract and a time trend. See the appendix for a detailed explanation of its construction. The estimated equilibrium strategies are strictly increasing satisfying C3 of

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Proposition 1. This suggests that the model is supported by the data, i.e. bidders behave according to the model. A striking feature of Figure 4 is that we do not observe any difference between the two equilibrium strategies. Both strategies perfectly superimpose suggesting that bidders adopt the same bidding strategy regardless of their type. Thus, firms are symmetric on eastern tracts. Results are quite different on western tracts, as shown by Figure 3, where we observe different bidding strategies. At a given cost, a weak bidder has a lower bid. This result is consistent with Maskin and Riley (2000a) in the sense that a weak bidder has to bid more aggressively to compensate his cost disadvantage to have a chance of winning the auction. This difference tends to decrease when firms incur larger costs. When superimposing both figures, we observe that strong bidders adopt similar strategies on western or eastern tracts while weak bidders tend to submit higher bids on western tracts than on eastern tracts at a given cost.

Table 4 provides some summary statistics on estimated costs. We observe a small difference of about Can$0.69 per meter or 6.5% between the two firms' types on western tracts. Given the intense competition in these auctions, this difference may create some asymmetry. We observe similar features for eastern tracts despite the differences in Figures 3 and 4. Figures in Table 4 are similar for strong and weak bidders regardless of the tract location. Table 5 provides some summary statistics on the winners' costs. The figures in Table 5 have similar interpretations as those in Table 4. Strong winning firms have costs that are about 8% lower than weak winning firms with no significant difference between western and eastern tracts. Summary statistics on the rents in value and in percentage left to winning firms are given in Table 6. We observe that informational rents are on average quite low. This suggests that, (i) these auctions and the market in general are very competitive and, (ii) the auction mechanism is quite successful in capturing firms' rents. As a matter of fact, the bids are very close to the costs, leaving firms with a small profit. Moreover, the City sets a low reserve price rendering the bidding more aggressive and hence, lowering the bids on average. On western tracts, the rents differ with the firms' type. Rents of weak winning firms are smaller by Can$0.30 per meter or 41% than those of strong firms. In percentage, the rent for strong winning firms is about twice as much as the rent for weak winning firms. This confirms the presence of potential asymmetry, in the sense that weak firms have to sacrifice a substantial amount of profit or rent to win these auctions because of their relative cost disadvantage. Table 6 clearly shows that

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on western tracts, weak winning firms can expect a lower profit than the strong winning firms. In eastern tracts, we observe similar features, but the differences between strong and weak firms are much less significant with a difference in rents equal to Can\$0.15.

To summarize, Figures 3 and 4 provide some support for asymmetry in western tracts, while in eastern tracts asymmetry among bidders seems to be almost nonexistent. Tables 4 and 5 provide some mitigated results in the sense that some evidence of asymmetry is found for both types of tracts. Table 6 provides support for asymmetry in western tracts, while bidding in eastern tracts seems to show little asymmetry. Considering the median of z, Figure 5 displays the densities for weak and strong bidders in western tracts, while Figure 6 displays their counterparts for eastern tracts. We observe unimodal densities with a symmetric shape for western tracts and a slightly skewed one for eastern tracts. The mode takes a larger value for the weak bidders than for the strong bidders for western tracts. It illustrates that weak bidders tend to draw larger costs than their strong opponents. The estimated cost densities confirm the presence of asymmetry on western tracts, while the asymmetry is almost nonexistent on eastern tracts.

4.2 A Policy Analysis

We focus on western tract auctions as these auctions display some asymmetry among bidders. Asymmetry among bidders is known to render auctions inefficient. When comparing the firm with the lowest cost with the identity of the winner for each auction, we find seven auctions in western tracts for which the lowest cost firm does not win. This represents about 24% of auctions. On average, the difference between the winning firm's cost and the most efficient firm's cost is equal to Can\$0.607 or 6.58% relative to the most efficient firm's cost.

Though first-price sealed-bid auctions are commonly used in procurements, we simulate an ascending auction. The policy experiments are performed as follows. Using the estimated bidders' costs, we compute their bidding strategies in different auction mechanisms and report the obtained results. We find that an ascending auction would reduce the total cost for allocating the contracts by 0.32%, which is quite small. A second simulation exercise consists in implementing the optimal mechanism in Section 3. The latter involves the computation of discriminatory optimal reserve prices, which requires the knowledge of c_0, i.e. the cost to clear the snow by the municipal employees. We consider c_0 to be

equal to the actual (observed) reserve price. We explain this choice as follows. Because of the truncation on the bid distributions, we can identify the cost distributions on $[\underline{c}, p_0]$ only. Considering $c_0 > p_0$ would require the estimation of $F(\cdot|\cdot)$ on $[p_0, \overline{c}]$. Consequently, we need to consider $c_0 \leq p_0$. The reserve price reflects the expected performance for a private firm with an average efficiency. In this case, as we cannot expect the public sector to do better than an average private firm, it is wise not to consider c_0 smaller than p_0. Therefore, we choose c_0 equal to p_0. For strong bidders, the so-called optimal discriminatory reserve price would be on average equal to Can\$12.38 per meter, while it would be on average equal to Can\$12.39 for weak bidders. The actual reserve price is equal on average to Can\$12.45. We then compute the virtual costs or $J_i(\cdot), i = 0, 1$ and compute the payments. We obtain that 57% of the western tracts would be won by weak firms, while only 38% of the western tracts are actually won by weak firms. Thus the optimal design increases the probability of winning for a weak firm. The cost reduction with the optimal design would be only 1.18%.

Since the optimal design is not implementable in practice, we consider a third policy experiment in the spirit of a price-preference policy with a subsidy to be given to weak bidders to enhance competition on strong bidders. The difference between c_{0W} and c_{1W} estimates the amount of subsidy for given tract characteristics. We estimate the expected cost value for strong firms $E(c_{1W}|z)$ and for weak bidders $E(c_{0W}|z)$ using kernel regression estimators. The subsidy is computed as the difference, i.e. $\hat{d}(z) = \hat{E}(c_{0W}|z) - \hat{E}(c_{1W}|z)$. It varies from Can\$0.00 to Can\$1.06 per meter with a mean equal to Can\$0.65 and a standard deviation equal to Can\$0.29. The policy should be announced prior to the auction. An eastern firm's cost would be $\tilde{c}_{0W} = c_{0W} - \hat{d}(z)$. Thus the game becomes symmetric as $F_{0W}(\tilde{c}_{0W})$ becomes similar to $F_{1W}(c_{1W})$. A symmetric equilibrium strategy is then used to compute the bids. This policy experiment is in the spirit of recent findings by Cantillon (2004). If the procurement agency subsidies some bidders, it should do so to generate the most symmetric game as asymmetries among bidders hurt the procurement costs.

We consider three scenarios while taking into account the subsidy cost for the City with the same level of competition. A subsidy policy would increase the probability of winning the auctions for weak firms as our results show that 61% of auctions would be won by weak firms. A first possibility for the City would be to provide a free storage

space to weak firms in lieu of a subsidy. After discussion with various firms and their professional association, we find that the rental cost in west Montréal represents the major cost difference among firms. Assuming that the City has such a space available and that the opporutnity cost is zero, i.e. there is no rental value, such an implicit subsidy would allow a cost reduction of 5.82% representing more than half a million of Canadian dollars. This scenario, which looks rather optimistic, is provided as a benchmarck since the western part of Montréal is quite urbanized. In a second scenario, we consider that the City pays the subsidy to the weak winning firm but we ignore the cost of public funds. Another possible interpretation is as follows. The City provides free storage space to firms and the implicit subsidy can be interpreted as the opportunity cost incurred by the City. The cost reduction would be about 2.37%, which represents about a quarter of a million of Canadian dollars. A third scenario extends the second scenario with a cost of public funds equal to 0.3, which is a standard estimate for western economies. Raising additional taxes to subsidize weak firms creates a distortion to the local economy, i.e. Can\$1 of subsidy really costs Can\$1.3. The costs would be reduced by 1.34%. We expect that such a policy will increase firms' participation in a long run perspective thereby increasing the level of competition. All these results should be considered cautiously. As a matter of fact, we could expect the cost reduction with the optimal design to be superior to those with subsidy policies. Note that the optimal auction design is based on a expected procurement cost. Thus the empirical validation of this optimality result on data would have required a large number of auctions.

5 Concluding Remarks

This paper makes several contributions to the empirical analysis of procurement auctions. A convenient estimation procedure is proposed to estimate asymmetric sealed-bid auctions with a binding reserve price. Because the structural approach allows to recover the informational structure of the model, the extent of inefficiency due to asymmetry is empirically assessed and various policy experiments such as an ascending auction, an optimal mechanism design and a sealed-bid auction with a subsidy to weak firms are conducted. Our results show that one can expect to decrease the costs for awarding the contracts with different magnitudes. Even though some of these results are predicted by the theoretical literature, this paper proposes their first empirical validation on field data.

In this respect, the methodology developed in this paper can be used to assess the gain of preference policies such as affirmative action and national buying policies. This represents a wide range of studies that empirical researchers and policy makers can now undertake.

Given the relatively small size samples usually available to researchers, parametric methods could be used for estimating bid and cost distributions. In our case, nonparametric estimators give good results, as the number of bids overcomes the low number of auctions. The auction model does not take into account the public nature of the auctioneer. We assume that its objective is to minimize the costs since municipalities generally have tight budget constraints. The policy experiments could indicate even greater benefits if we were assuming welfare maximizing municipalities. The cost savings could be used for other public services, which in turn would improve population welfare.

Appendix

Proof of Proposition 1: We provide a sketch of the proof.

First, we show the necessity of C1, C2 and C3. Condition C1 must hold since a potential bidder bids if and only if his cost is at most p_0. Thus $0 < F_0(p_0) < 1$ and $0 < F_1(p_0) < 1$ follow from $p_0 \in (\underline{c}, \overline{c})$. Regarding C2, $b_{1j} = 0$ if $c_{1j} > p_0$ for $j = 1, \ldots, n_1$ and $b_{0j} = 0$ if $c_{0j} > p_0$ for $j = 1, \ldots, n_0$. For any $(b_1, \ldots, b_j) \in I\!R_+^j$, using that the b_{ij}s are iid and $n_1^* \sim B(n_1, F_1(p_0))$, $n_0^* \sim B(n_0, F_0(p_0))$, we have $\Pr(b_{11}^* \leq b_1, \ldots, b_{1j}^* \leq b_i | n_1^* = j, n_1, p_0) = \prod_{r=1}^{j}[F_1(s_1^{-1}(b_r))/F_1(p_0)]$ and $\Pr(b_{01}^* \leq b_1, \ldots, b_{0j}^* \leq b_i | n_0^* = j, n_0, p_0) = \prod_{r=1}^{j}[F_0(s_0^{-1}(b_r))/F_0(p_0)]$. Because $s_1(p_0) = s_0(p_0) = p_0$, $F_0(p_0) > 0$ and $F_1(p_0) > 0$, $\lim_{c\uparrow p_0} s_1'(c)/f_1(c) = \lim_{c\uparrow p_0} s_0'(c)/f_0(c) = 0$. Thus, it can be easily shown that $\lim_{b\uparrow p_0} g_1^*(b) = \lim_{p\uparrow p_0} g_0^*(b) = +\infty$.

Regarding C3, let $s_1(\cdot)$ and $s_0(\cdot)$ be the strictly increasing differentiable equilibrium strategies corresponding to $F_1(\cdot)$ and $F_0(\cdot)$ with support $[\underline{c}, \overline{c}]$. Let $G_1^*(\cdot)$ and $G_0^*(\cdot)$ be the distributions defined by $G_1^*(b) = F_1(s_1^{-1}(b, F_0, F_1, n))/F_1(p_0)$ and $G_0^*(b) = F_0(s_1^{-1}(b, F_0, F_1, n))/F_0(p_0)$ for every $b \in [\underline{b}, \overline{b}]$. Now, $s_1(\cdot)$ and $s_0(\cdot)$ must solve the system of differential equations (3)-(4). But, because (5)-(6) follow from (3)-(4), $s_1(\cdot)$ and $s_0(\cdot)$ must satisfy $\xi_1(s_1(c_1, F_0, F_1, n), G_0^*, G_1^*, n) = c_1$ and $\xi_0(s_0(c_0, F_0, F_1, n), G_0^*, G_1^*, n) = c_0$ for any $c_0, c_1 \in [\underline{c}, \overline{c}]$. We then obtain $\xi_1(\cdot, G_0^*, G_1^*, n) = s_1^{-1}(\cdot, F_0, F_1, n, p_0)$ on $[\underline{b}, p_0]$, and $\xi_0(\cdot, G_0^*, G_1^*, n) = s_0^{-1}(\cdot, F_0, F_1, n, p_0)$ on $[\underline{b}, p_0]$. Hence, C3 must hold because $s_1^{-1}(\cdot)$ and $s_0^{-1}(\cdot)$ are strictly increasing on $[\underline{b}, p_0]$.

Second, we show the sufficiency of C1, C2 and C3. Let us choose $p_0 < \overline{c}$, $0 < F_0(p_0) < 1$, $0 < F_1(p_0) < 1$. Using the distributions $G_0^*(\cdot)$ and $G_1^*(\cdot)$ on $[\underline{b}, p_0]$ and $F_1^*(\cdot) = F_1(\cdot)/F_1(p_0)$, $F_0^*(\cdot) = F_0(\cdot)/F_0(p_0)$ defined on $[\underline{c}, p_0]$, we can easily extend the proof of Theorem 4 in Guerre <u>at al.</u> (2000). The conditions $\lim_{b\uparrow p_0} \xi_1(b, G_0^*, G_1^*, n, F_0(p_0), F_1(p_0)) = p_0$ and $\lim_{b\uparrow p_0} \xi_0(b, G_0^*, G_1^*, n, F_0(p_0), F_1(p_0)) = p_0$ follow from (5) and (6) and $\lim_{b\uparrow p_0} g_0^*(b) = \lim_{b\uparrow p_0} g_1^*(b) = +\infty$.

If C1–C3 hold, we can establish that there exist two distributions $F_0(\cdot)$ and $F_1(\cdot)$ on $[\underline{c}, \overline{c}]$ with $\underline{c} = \xi_1(\underline{b}, G_0^*, G_1^*, n, F_0(p_0), F_1(p_0)) = \xi_1(\underline{b}, G_0^*, G_1^*, n, F_0(p_0), F_1(p_0))$ continuous on $[\underline{c}, p_0]$ and rationalizing $G_0^*(\cdot)$ and $G_1^*(\cdot)$ in a first-price sealed-bid auction with p_0. Distributions $F_1(\cdot)$ and $F_0(\cdot)$ can be obtained by extension in an absolute continuous fashion to the interval $[p_0, \overline{c}]$. To prove the last part of Theorem 1, note

that n_1 and $F_1(p_0)$ are identified from $\pi_1(\cdot)$, while n_0 and $F_0(p_0)$ are identified from $\pi_0(\cdot)$. Moreover, we have $\xi_1(\cdot, G_0^*, G_1^*, n, F_0(p_0), F_1(p_0)) = s_1^{-1}(\cdot, F_0, F_1, n, p_0)$ on $[\underline{b}, p_0]$, $\xi_0(\cdot, G_0^*, G_1^*, n, F_0(p_0), F_1(p_0)) = s_0^{-1}(\cdot, F_0, F_1, n, p_0)$ on $[\underline{b}, p_0]$ and the truncated distributions $F_1^*(\cdot) = F_1(\cdot)/F_1(p_0)$ and $F_0^*(\cdot) = F_0(\cdot)/F_0(p_0)$ are unique and equal to $G_1^*(\xi_1^{-1}(\cdot, G_1^*, G_0^*, n, F_1(p_0), F_0(p_0)))$ and $G_0^*(\xi_0^{-1}(\cdot, G_1^*, G_0^*, n, F_1(p_0), F_0(p_0)))$ on $[\underline{c}, p_0] = [\xi_1(\underline{b}, G_0^*, G_1^*, n, F_0(p_0), F_1(p_0)), p_0] = [\xi_0(\underline{b}, G_0^*, G_1^*, n, F_0 \ (p_0), F_1(p_0)), p_0]$, respectively. Because n_1, n_0, $F_0(p_0)$, $F_1(p_0)$ are identified, it follows that $F_1(\cdot)$ and $F_0(\cdot)$ are uniquely determined on $[\underline{c}, p_0]$ and equal to $F_1(p_0)G_1^*(\xi_1^{-1}(\cdot, G_1^*, G_0^*, n, F_1(p_0), F_0(p_0)))$ and $F_0(p_0)G_0^*(\xi_1^{-1}(\cdot, G_1^*, G_0^*, n, F_1(p_0), F_0(p_0)))$ on this interval, respectively.□

A Simulation Study of a Subsidy Policy

Public institutions sometimes give a preference to a local firm or a firm owned by minorities under the form of a discounted bid. The question to subsidize non competitive firms is in the core of policy debates and is often wrongly assumed to be costly. Simulation studies by McAfee and McMillan (1989) and Harstad et al. (2003) show that a price-preference policy reduces the costs. Relying on experimental data, Corns and Schotter (1999) obtain similar results. We conduct a simulation study. Unlike the previous literature, we use an indirect approach based on observed bids $(b_{ij}, i = 1, 0, j = 1, \ldots, n_i)$ generated from sealed-bid auctions. To simplify, we consider no reserve price and homogeneous auctioned projects. The bids are assumed to be independent and distributed uniformly as $G_1(\cdot)$ and $G_0(\cdot)$ on $[7, 13]$ and $[9, 15]$, respectively. There are 6 strong bidders and 11 weak bidders. These values are consistent with the bid data on western tracts. See Sections 2 and 4.

We make independent draws for 30 auctions, which gives a total of 180 bids for strong bidders and 330 bids for weak bidders. We recover the bidders' costs using the inverse bidding strategies. With uniform bid distributions, this gives

$$c_{1i} = b_{1i} - \frac{1}{\frac{5}{13-b_{1i}} + \frac{11}{15-b_{1i}}}, i = 1, \ldots, 6, \quad c_{0i} = b_{0i} - \frac{1}{\frac{6}{13-b_{0i}} + \frac{10}{15-b_{0i}}}, i = 1, \ldots, 11.$$

The latter clearly shows that for any b, the weak bidder will have a larger cost than the strong bidder. Similarly for any c, the weak bidder will bid more aggressively than the strong bidder. This difference may lead to some inefficiency though both strategies are strictly increasing. Because of uniform distributions, the extent of inefficiency reduces to

less than a cent, while in Section 4 inefficiency is more important because of the skewed bid distributions. Following the previously defined inverse bidding strategies, the strong firm bids 9.100 with a cost at 8.782 and the weak firm bids 9.098 with a cost at 8.788. The weak firm will win the auction though its cost is slightly larger than the one of its (strong) opponent.

Using the recovered costs, we estimate nonparametrically the cost densities $f_1(\cdot)$ and $f_0(\cdot)$. We compute the difference $d = \mathrm{E}(c_0) - \mathrm{E}(c_1)$. We find it equal to 2.56. This difference is the estimate of the subsidy to be given to the winning weak bidder. The value d approximates the shift between the two densities $f_1(\cdot)$ and $f_0(\cdot)$. Consequently, the weak bidder will draw a cost $\tilde{c}_0 = c_0 + d$. The game becomes symmetric, which greatly simplifies the simulation. This subsidy has the main effect to induce strong bidders to bid more aggressively as they will have to bid against 15 "real" competitors instead of 5 previously. The benefit of this additional competitive pressure on strong bidders is expected to outweigh the cost to subsidize a weak winning bidder.

Let $s(\cdot)$ denote the symmetric equilibrium strategy, namely $s(c) = c + \{\int_c^{\overline{c}}[1-F_1(y)]^{n_1+n_0-1} \, dy / [1-F_1(c)]^{n_1+n_0-1}\}$. The expected cost to allocate the 30 contracts becomes

$$30 \times \int_{\underline{c}}^{\overline{c}} \left[\frac{n_1}{n_0+n_1} s(y) + \frac{n_0}{n_1+n_0}[s(y)+d] \right] dF_{\min}(y),$$

where $F_{\min}(\cdot)$ is the distribution of the minimum cost, $[\underline{c}, \overline{c}] = [s_1^{-1}(7), s_1^{-1}(13)]$ in a symmetric game with a cost distribution $F_1(\cdot)$. The weak bidders are expected to win $11/17$ or about 65% of the subsidized auctions, which involves an additional subsidy cost for the City denoted d. We find that the expected cost for allocating contracts would decrease by about 25%. Note that this subsidy policy is in the spirit of Affirmative Action in the sense that it gives equal chance of winning to all bidders.

Ayres and Cramton (1996) have noted that subsidizing weak bidders can also promote entry of new bidders, especially the weak ones in a long run perspective. Such a possibility has been also studied by Harstad et al. (2003) for a common value auction model. Assuming that the number of strong firms remains the same ($n_1 = 6$), while the number of weak firms increases by 2 ($n_0 = 13$), we find that the expected cost would be reduced by 6%. Because of the independence of private values, increasing competition reduces bids. When considering other frameworks such as affiliated private values or common value, increasing competition may have an opposite effect on bids.

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Some Practical Issues

Construction of the Variable z_ℓ: Because of the relatively small size of our data set and the use of nonparametric estimators, we choose to construct a single variable z_ℓ to capture auction heterogeneity. We use a principal component analysis. Three variables characterize the auctioned tracts, namely the distance to the snow dump, the length of the tract and a time trend capturing the technological progress over the period. All of them are highly correlated with observed bids and reserve prices. We construct the following variable $z_\ell = 0.599 L_\ell + 0.464 D_\ell + 0.652 t_\ell$, where L_ℓ, D_ℓ and t_ℓ are the standardized variables for length, distance and time trend, respectively. The variable z_ℓ is of dimension one, i.e. $d = 1$. We obtain z_ℓ varying from -2.44 to 2.31 with a mean equal to 0 and a variance equal to 1.17 for the 61 auctions. Because the tract location is also another important tract characteristic, we estimate the model for eastern and western tracts, obtaining estimates for $\xi_{1W}(\cdot)$, $\xi_{0W}(\cdot)$, n_{1W}, n_{0W}, $f_{1W}(\cdot|\cdot)$ and $f_{0W}(\cdot|\cdot)$ for western tracts, and $\xi_{1E}(\cdot)$, $\xi_{0E}(\cdot)$, n_{1E}, n_{0E}, $f_{1E}(\cdot|\cdot)$ and $f_{0E}(\cdot|\cdot)$ for eastern tracts.

Choice of Kernels and Bandwidths: Among the large number of differentiable kernels defined on compact support, we choose the biweight kernel $K(u) = (15/16)(1-u^2)^2 \, 1\!\!1(|u| \leq 1)$. This kernel is used for $K_G(\cdot)$ and $K_z(\cdot)$, while $K_g(\cdot, \cdot)$ and $K_f(\cdot, \cdot)$ are the products of two univariate biweight kernels. The choice of bandwidth requires, however, more attention. A too large bandwidth tends to oversmooth the estimated density, while a too small bandwidth tends to undersmooth the estimated density affecting the bias and variance of the estimator. Let L_W and L_E be the number of auctions on western and eastern tracts. By doing separate estimations for western and eastern tracts, we have a total of 14 bandwidths. We assume that the densities $f_1(\cdot)$ and $f_0(\cdot)$ admit R bounded continuous derivatives. Because the kernel function has to be of order $R + 1$, we consider $R = 1$. Following Guerre <u>et al.</u> (2000), the bandwidths are of the following form for the first and second step of our estimator $h_G = c_G(nL)^{-1/5}$, $h_g = c_g(nL)^{-1/6}$, $h_z = c_z(L)^{-1/5}$, $h_f = c_f(nL)^{-1/6}$, where c_G, c_g, c_z and h_f are some constants and nL denotes the total number of bids or costs.

The total number of bids or costs nL varies for eastern or western tracts and for firms of type 1 or 0. Specifically, $nL_{1W} = 80$, $nL_{0W} = 116$, $nL_{1E} = 150$ and $nL_{0E} = 111$. The number L for h_z will vary for eastern or western tracts. The constants are determined

by the so-called rule of thumb. Namely, the constants are equal to $2.623 \times 1.06\hat{\sigma}$, where $\hat{\sigma}$ is the empirical standard deviation of observations. The factor 2.623 is a correction due to the use of a biweight kernel instead of a Gaussian kernel. See Hardle (1991). We find $h_{G1W} = 1.54$, $h_{G0W} = 1.52$, $h_{g1W} = 2.14$, $h_{g0W} = 1.71$, $h_{zW} = 1.71$, $h_{f1W} = 2.10$, $h_{f0W} = 1.67$, $h_{G1E} = 1.75$, $h_{G0E} = 1.75$, $h_{g1E} = 1.68$, $h_{g0E} = 1.99$, $h_{zE} = 1.96$, $h_{f1E} = 1.81$, $h_{f0E} = 1.88$.

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